Saturday, August 02, 2014
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Why I like General Relativity (GR):

- GR is one of the few "brand new" subjects that students see late in the game (after they have a pretty solid understanding of basic physics).
- GR raises/addresses some of the BIG qustions in physics, e.g. cosmology, nature of spacetime.
- GR is ½ of the most perplexing issue confronting theoretical physics, i.e. Quantum Gravity.

What is "general" about GR and how is it related to Special Relativity (SR)?
Many people think that GR is a generalization of SR... and they are wrong! (but really, who could blame them?!)

To answer this we will first consider familiar correspondence principles (general → limited).

$\boxed{\text{Newtonian Mechanics}}$ (most limited case) (particles + fields)

$S \gg \hbar$

$\boxed{\text{NR Quantum Mechanics}}$ (particles + fields)

slow speeds compared to c

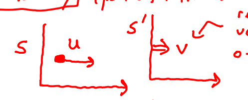
$\boxed{\text{Special Relativity}}$ (particles + fields)

Bohr: QM is true for large systems
but the discreteness is relatively small.

e.g. SHO $E_n = (n+\frac{1}{2})\hbar\omega$
$E = \frac{1}{2} m\omega^2 A$
$m = 1\,\text{kg}$, $\omega = 1\,\text{Hz}$, $A = 1\,\text{m}$

$n \sim 4.7\times 10^{33}$
$\Downarrow$
$\frac{\Delta E}{E} \sim 0$

Velocity addition:



relative velocity of S + S'

velocity seen in S'

velocity seen in S

$$u' = \frac{v+u}{1+vu/c^2} \Rightarrow v \text{ or } u = c \Rightarrow u' = c$$

For $vu/c^2 \ll 1$

$u' \sim v + u$ (Galilean v.a.)

Statistical: Classical behavior arises for many particles due to lack of wavefunction coherence (exceptions are condensates)

Path Integral:
Path weighted by $e^{\frac{i}{\hbar}\int \overbrace{\frac{1}{2}mv^2}^{L_{free}} dt}$, so when $\hbar \ll S = \int L\,dt$
rapid fluctuations cancel except for extrema of $\int \frac{1}{2}mv^2 dt$, i.e. classical motion.

$S \gg \hbar$

"$v \ll c$"

$\boxed{\text{Quantum Field Theory}}$

no more particles, only fields

Why a field theory?

at odds w/ each other
- QM for particles requires wavefunction normalization to conserve particle number.
- SR for particles allows creation/annihilation or changing particle number.

We could combine them w/out fields, but it is ugly and doesn't incorporate effects that we know happen, e.g. Higgs mechanism.

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Okay, so where does GR fit in? All of the above (NM, QM, SR, QFT) are frameworks for mechanics. By themselves they do no constitute theories. To have a theory you need a framework plus degrees of freedom and their interactions.

Punchline: GR is a theory, not a framework. It is a theory of the gravitational interaction.

Two conclusions: a) There is no correspondence principle relating GR to SR.

b) GR is actually a generalization of Newtonian gravity (needed when $\frac{M}{R} \gtrsim \frac{c^2}{G}$).

Okay, but you still might ask how GR and SR are related. The answer is that SR is one particular solution of GR, i.e. a flat spacetime w/ no gravity acting.

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In going beyond Newtonian gravity to GR we are going to be forced to reckon w/ a lot of new ideas, e.g. manifolds, curvature, tensors, etc.

Before it gets hairy though, we can paraphrase GR by analogy with another theory you know well,

Electrodynamics:

$$\left.\begin{array}{l}\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\epsilon_0}\\ \vec{\nabla}\times\vec{B}=\mu_0\vec{J}+\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\end{array}\right\}\text{Field Eqns.}\qquad \left.\begin{array}{l}\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\\ \vec{\nabla}\cdot\vec{B}=0\end{array}\right\}\text{BI}$$

Maxwell's Eqns. $\Rightarrow$ Tell us how sources $(\rho,\vec{J})$ create fields $(\vec{E},\vec{B})$ (w/ some topological constraints)

$$\left.\begin{array}{ll}\vec{F}_{em}=q(\vec{E}+\vec{v}\times\vec{B}) & \text{Lorentz Force}\\ \sum_i \vec{F}_{ib}=m_b\vec{a}_b & \text{N2L}\end{array}\right\}$$ Tell us how particles react to the fields.

This is an example of a background/test particle split which though not fundamental, is still very useful for computation and application!

General Relativity:

Einstein's Equation $\longrightarrow$ Tells us how sources create gravitational fields/curvature.

Geodesic Equation $\longrightarrow$ Tells us how a test particle responds to curvature.

So do we have a gravitational field? Actually no, not in the Newtonian sense. However, in describing the curvature of a spacetime we will need to introduce perhaps the single most important element of GR, the metric field.